USING THE QUADRATIC FORMULA 9.1.2 and 9.1.3

When a quadratic equation is not factorable, another method is needed to solve for x. The Quadratic Formula can be used to calculate the roots of a quadratic function, that is, the x-intercepts of the parabola. The Quadratic Formula can be used with any quadratic equation, factorable or not. There may be two, one, or no solutions, depending on whether the parabola intersects the x-axis twice, once, or not at all.
The solution(s) to any quadratic equation $ax^2 + bx + c = 0$ are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The $\pm$ symbol is read as "plus or minus." It is shorthand notation that tells you to calculate the formula twice, once with + and again with – to get both x-values.

To use the formula, the quadratic equation must be written in *standard form*: $ax^2 + bx + c = 0$. This is necessary to correctly identify the values of a, b, and c. Once the equation is in standard form and equal to 0, a is the coefficient of the x^2 term, b is the coefficient of the x-term and c is the constant.

Example 1

Solve $2x^2 - 5x - 3 = 0$.

Identify a, b, and c. Watch your signs carefully. $a = 2, b = -5, c = -3$

Write the Quadratic Formula. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Substitute a, b, and c in the formula and do the initial calculations. $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$

$x = \frac{5 \pm \sqrt{25 - (-24)}}{4}$

Simplify the $\sqrt{\ }$. $x = \frac{5 \pm \sqrt{49}}{4}$

Calculate both values of x. $x = \frac{5+7}{4} = \frac{12}{4} = 3$ or $x = \frac{5-7}{4} = \frac{-2}{4} = -\frac{1}{2}$

The solution is $x = 3$ or $x = -\frac{1}{2}$.

Example 2

Solve $3x^2 + 5x + 1 = 0$.

Identify a, b, and c.	$a = 3,\ b = 5,\ c = 1$
Write the Quadratic Formula.	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Substitute a, b, and c in the formula and do the initial calculations.	$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(3)(1)}}{2(3)}$
	$x = \frac{-5 \pm \sqrt{25 - 12}}{6}$
Simplify the $\sqrt{\ }$.	$x = \frac{-5 \pm \sqrt{13}}{6}$

The solution is $x = \frac{-5+\sqrt{13}}{6} \approx -0.23$ or $x = \frac{-5-\sqrt{13}}{6} \approx -1.43$.

Example 3

Solve $25x^2 - 20x + 4 = 0$.

Identify a, b, and c.	$a = 25,\ b = -20,\ c = 4$
Write the Quadratic Formula.	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Substitute a, b, and c in the formula and do the initial calculations.	$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(25)(4)}}{2(25)}$
	$x = \frac{20 \pm \sqrt{400 - 400}}{50}$
Simplify the $\sqrt{\ }$.	$x = \frac{20 \pm \sqrt{0}}{50}$

This quadratic has only one solution: $x = \frac{2}{5}$.

Example 4

Solve $x^2 + 4x + 10 = 0$.

Identify a, b, and c.	$a = 1, b = 4, c = 10$
Write the Quadratic Formula.	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Substitute a, b, and c in the formula and do the initial calculations.	$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(10)}}{2(1)}$
	$x = \frac{-4 \pm \sqrt{16 - 40}}{2}$
Simplify the $\sqrt{\ }$.	$x = \frac{-4 \pm \sqrt{-24}}{2}$

It is impossible to take the square root of a negative number; therefore this quadratic has **no** real solution.

Example 5

Solve $(3x + 1)(x + 2) = 1$.

Rewrite in standard form.	$(3x + 1)(x + 2) = 1$
	$3x^2 + 7x + 2 = 1$
	$3x^2 + 7x + 1 = 0$
Identify a, b, and c.	$a = 3, b = 7, c = 1$
Write the quadratic formula.	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Substitute a, b, and c in the formula and do the initial calculations.	$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(3)(1)}}{2(3)}$
	$x = \frac{-7 \pm \sqrt{49 - 12}}{6}$
Simplify .	$x = \frac{-7 \pm \sqrt{37}}{6}$

Problems

Solve each equation by using the Quadratic Formula.

1. $x^2 - x - 2 = 0$
2. $x^2 - x - 3 = 0$
3. $-3x^2 + 2x + 1 = 0$
4. $-2 - 2x^2 = 4x$
5. $7x = 10 - 2x^2$
6. $-6x^2 - x + 6 = 0$
7. $6 - 4x + 3x^2 = 8$
8. $4x^2 + x - 1 = 0$
9. $x^2 - 5x + 3 = 0$
10. $0 = 10x^2 - 2x + 3$
11. $x(-3x + 5) = 7x - 10$
12. $(5x + 5)(x - 5) = 7x$

Answers

1. $x = 2$ or -1
2. $x = \frac{1 \pm \sqrt{13}}{2}$ ≈ 2.30 or -1.30
3. $x = -\frac{1}{3}$ or 1
4. $x = -1$
5. $x = \frac{-7 \pm \sqrt{129}}{4}$ ≈ 1.09 or -4.59
6. $x = \frac{1 \pm \sqrt{145}}{-12}$ ≈ -1.09 or 0.92
7. $x = \frac{4 \pm \sqrt{40}}{6} = \frac{2 \pm \sqrt{10}}{3}$ ≈ 1.72 or -0.39
8. $x = \frac{-1 \pm \sqrt{17}}{8}$ ≈ 0.39 or -0.64
9. $x = \frac{5 \pm \sqrt{13}}{2}$ ≈ 4.30 or 0.70
10. no solution
11. $x = \frac{2 \pm \sqrt{124}}{-6} = \frac{1 \pm \sqrt{31}}{-3}$ ≈ -2.19 or 1.52
12. $x = \frac{27 \pm \sqrt{1229}}{10}$ ≈ 6.21 or -0.81